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1. Linear ODE. If p and r in $y' + p(x)y = r(x)$ are continuous for all x in an interval $|x - x_0| \leq a$, show that $f(x, y)$ in this ODE satisfies the conditions of our present theorems, so that a corresponding initial value problem has a unique solution. Do you actually need these theorems of this ODE?

3. Vertical strip. If the assumptions of theorems 1 and 2 are satisfied not merely in a rectangle but in a vertical infinite strip $|x - x_0| < a$, in what interval will the solution of (1) exist?

5. Length of x -interval. In most cases the solution of an initial value problem (1) exists in an x -interval larger than that guaranteed by the present theorems. Show this fact for $y' = 2y^2$, $y(1) = 1$ by finding the best possible a (choosing b optimally) and comparing the result with the actual solution.

7. Maximum α . What is the largest possible α in example 1 in the text?

9. Common points. Can two solution curves of the same ODE have a common point in a rectangle in which the assumptions of the present theorems are satisfied?